

Divisible left-residuated posets, semilattices and Heyting algebras with operators

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Starting from structures slightly more general than partially ordered left-residuated integral monoids (polrims), we show that the divisibility axiom is equivalent to two identities (even in the absence of associativity). We then investigate the algebraic structure of left-residuated semilattices and generalized hoops using poset-product decompositions. This leads to an elementary way to calculate homomorphic images of finite residuated lattices as relativizations with respect to negative central idempotent elements. As an application we give a description of congruences in residuated Heyting algebras, with an extension to Heyting algebras with operators. A result obtained jointly with N. Galatos proves the finite model property for residuated Heyting algebras and any subvarieties determined by so-called simple rules. This shows that the corresponding equational theories are decidable.