

Sheaf representations via duality

Mai Gehrke

Université Paris Diderot, France
mgehrke@liafa.univ-paris-diderot.fr

This talk mainly focuses on joint work with Sam van Gool in which we exhibit a correspondence between soft sheaf representations of universal algebras and frame homomorphisms into subframes of their congruence lattices. For distributive-lattice-ordered algebras this allows us to dualise such sheaf representations.

An important feature, that is essential for applications, is that we allow the base spaces of the sheaves we consider to be non-Hausdorff. On the other hand, for our results to go through we require a tight relationship between the open and the compact sets of these spaces. A natural class of spaces, whose features are particularly well adapted to our results, are the stably compact spaces. This is a class of topological spaces closely related to Nachbin's compact ordered spaces, which provides a common generalization of compact Hausdorff spaces and spectral spaces. Stably compact topologies naturally come with an associated dual topology; the two topologies are related by being the open up-sets and down-sets, respectively, of the topology of a compact ordered space.

This so-called co-compact duality for stably compact spaces plays a prominent role in our main result: soft sheaf representations of an algebra over a stably compact base space correspond bijectively to frame homomorphisms from the open set lattice of the co-compact dual of the base space to a frame of permuting congruences of the algebra.

Our main application of this result is that soft sheaf representations of distributive lattices correspond bijectively to continuous decompositions of its Priestley dual space which satisfy an 'interpolation' property that we introduce. As a further application we obtain simple proofs of previously known results on sheaf representations of MV-algebras and point the way towards possible generalisations for integral and commutative GBL algebra (ongoing work joint with Peter Jipsen and Anna Carla Russo).